Cumulative Types Systems and Levels

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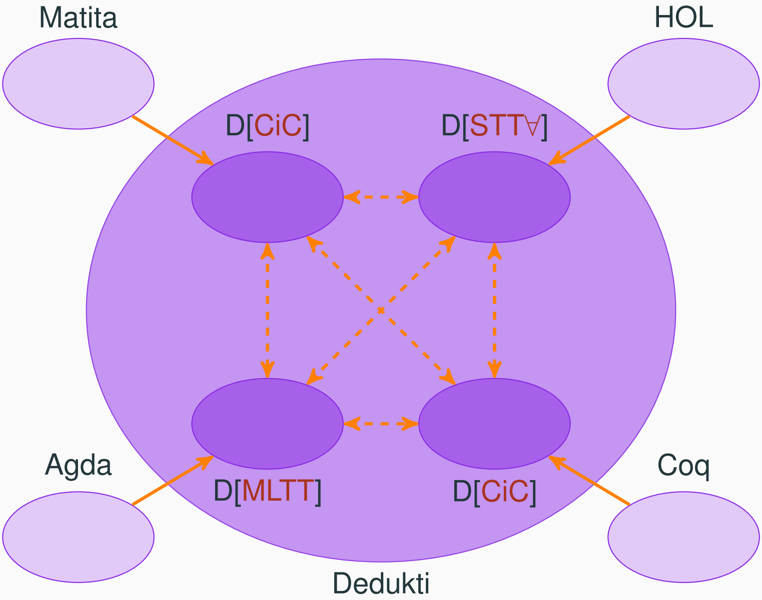

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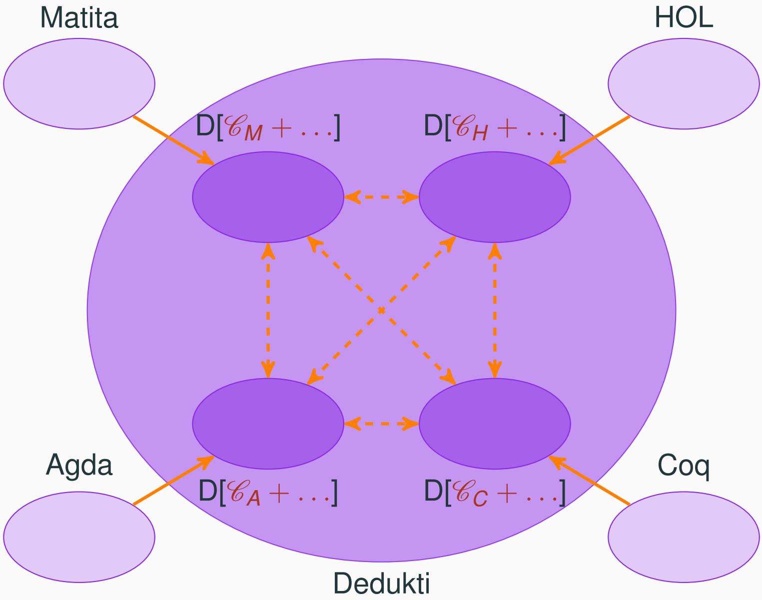

Cumulative Type Systems

$$\mathscr{C} = (\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{C})$$

Sorts — Axioms — Rules — Cumul.

Syntax

$$t, u, A, B ::= s \in \mathcal{S} \ \mid \ x \ \mid \ t\ u \ \mid \ \lambda x : A.\, t \ \mid \ (x : A) \to B$$

$$\frac{\Gamma \vdash_{\mathscr{C}} A : s_1 \qquad \Gamma, x : A \vdash_{\mathscr{C}} B : s_2 \qquad (s_1, s_2, s_3) \in \mathcal{R}}{\Gamma \vdash_{\mathscr{C}} (x : A) \to B : s_3}\ \Pi$$

$$\frac{\Gamma \vdash^{o}_{\mathscr{C}} \textbf{wf} \qquad (s_1, s_2) \in \mathcal{A}}{\Gamma \vdash^{o}_{\mathscr{C}} s_1 : s_2}\ \mathscr{C}_{sort}$$

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \preceq^{\mathcal{C}}_{\mathscr{C}} B}{\Gamma \vdash_{\mathscr{C}} t : B}\ Conv(CTS)$$

Cumulative Type Systems

$$\mathscr{C} = (\mathcal{S}, \mathcal{A}, \mathcal{R}, \emptyset)$$

Sorts ($\mathcal{S}$), Axioms ($\mathcal{A}$), Rules ($\mathcal{R}$), Cumul. ($\emptyset$)

Syntax

$$t, u, A, B ::= s \in \mathcal{S} \ | \ x \ | \ t\ u \ | \ \lambda x : A.\, t \ | \ (x : A) \to B$$

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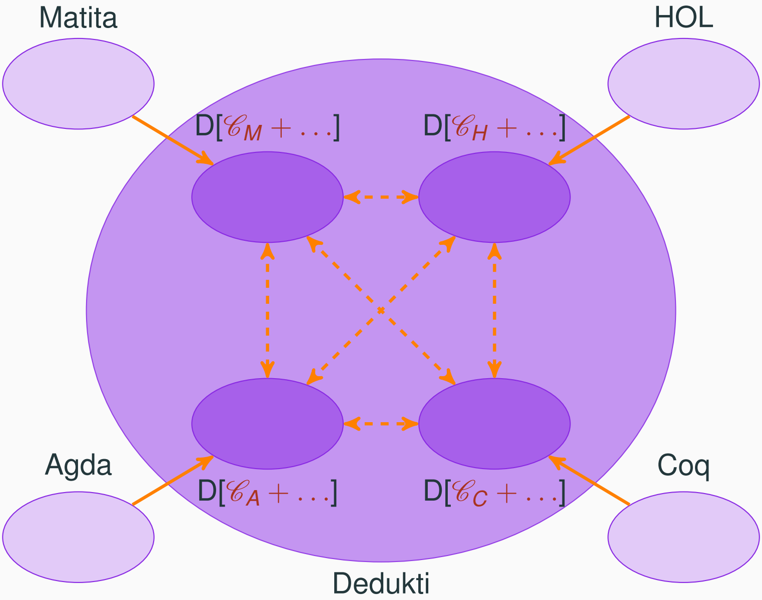
$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \equiv_{\beta} B}{\Gamma \vdash_{\mathscr{C}} t : B}\ Conv(PTS)$$

Translations



Strange loop 1

Correctness of the translation:

$$\Gamma \vdash_{\mathscr{C}} t : A \Rightarrow [\![\Gamma]\!] \vdash_{\mathscr{D}} [t] : [\![A]\!]$$

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Main lemma:

1. $A \equiv_\beta B \Rightarrow [\![A]\!] \equiv_\beta [\![B]\!]$

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Main lemma:

1. $A \equiv_\beta B \Rightarrow [\![A]\!] \equiv_\beta [\![B]\!]$
2. $[t]\,\{x \leftarrow [N]\} = [t\,\{x \leftarrow N\}]$

Dependencies:

- $1 \rightarrow 2$

Strange loop 1

Correctness of the translation:

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2. $[t]\,\{x \leftarrow [N]\} = [t\,\{x \leftarrow N\}]$

Dependencies:

- $1 \rightarrow 2$
- $2 \rightarrow 1$

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \equiv_\beta B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \mathit{Conv}$$

Strange loop 1

Correctness of the translation:

$$\Gamma \vdash_{\mathscr{C}} t : A \Rightarrow [\![\Gamma]\!] \vdash_{\mathscr{D}} [t] : [\![A]\!]$$

Main lemma:

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Dependencies:

- $1 \rightarrow 2$
- $2 \rightarrow 1$ but for the type

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \equiv_\beta B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \mathit{Conv}$$

Expansion Postponement

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad A \equiv_\beta B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \textit{Conv}$$

$$\swarrow \qquad \searrow$$

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad A \hookrightarrow_\beta^* B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \textit{Red} \qquad \frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad A \hookleftarrow_\beta^* B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \textit{Exp}$$

Expansion postponement

$$\Gamma \vdash_{\mathscr{C}} t : A \Leftrightarrow \exists A', A \hookrightarrow_\beta^* A' \wedge \Gamma \vdash_{\mathscr{C}}^{r} t : A'$$

Expansion Postponement

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad A \equiv_{\beta} B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \textit{Conv}$$

$$\Downarrow$$

$$\frac{\Gamma \vdash^{r}_{\mathscr{C}} t : A \qquad A \hookrightarrow^{*}_{\beta} B}{\Gamma \vdash^{r}_{\mathscr{C}} t : B}\ \textit{Red}$$

Expansion postponement

$$\Gamma \vdash_{\mathscr{C}} t : A \Leftrightarrow \exists A', A \hookrightarrow^{*}_{\beta} A' \wedge \Gamma \vdash^{r}_{\mathscr{C}} t : A'$$

Strange Loop 2

Let's try to prove Expansion Postponement (abstraction case):

$$\frac{\Gamma, x : A \vdash_{\mathscr{C}} t : B \qquad \Gamma \vdash_{\mathscr{C}} (x : A) \to B : s}{\Gamma \vdash_{\mathscr{C}} \lambda x : A.\, t : (x : A) \to B}\ \lambda \qquad\qquad \frac{}{?}$$

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$$\frac{\Gamma, x : A \vdash^{r}_{\mathscr{C}} t : B' \qquad \Gamma \vdash^{r}_{\mathscr{C}} (x : A) \to B : s}{?}$$

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You need subject reduction for $\Gamma \vdash^{r}_{\mathscr{C}} t : (x : A) \to B$! But...

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You need subject reduction for $\Gamma \vdash^{r}_{\mathscr{C}} t : (x : A) \to B$! But...

1. Subject Reduction needs the substitution lemma
2. The substitution lemma needs subject reduction (for the same reason as above) on <u>the type</u>

Explicit conversion

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad A \equiv_{\beta} B}{\Gamma \vdash_{\mathscr{C}} t : B} \; Conv$$

$$\Downarrow$$

$$\frac{\Gamma \vdash^{e}_{\mathscr{C}} t : A \qquad \Gamma \vdash^{e}_{\mathscr{C}} A \equiv_{\beta} B : s}{\Gamma \vdash^{e}_{\mathscr{C}} t : B} \; Red$$

Explicit conversion

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad A \equiv_\beta B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \mathit{Conv}$$

$$\frac{\Gamma \vdash^e_{\mathscr{C}} N : A \qquad \Gamma, x : A \vdash^e_{\mathscr{C}} M : B \qquad \begin{array}{c}\Gamma \vdash^e_{\mathscr{C}} A : s_1 \qquad \Gamma \vdash^e_{\mathscr{C}} B : s_2 \\ (s_1, s_2, s_3) \in \mathcal{R}_{\mathscr{C}}\end{array}}{\Gamma \vdash^e_{\mathscr{C}} (\lambda x : A.\, M)\ N \equiv_\beta M\{x \leftarrow N\} : B\{x \leftarrow N\}}\ \mathscr{C}^{\equiv_\beta}_{\mathrm{beta}}$$

$$\ldots$$

Explicit conversion

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad A \equiv_{\beta} B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \textit{Conv}$$

$$\Downarrow$$

$$\frac{\Gamma \vdash^{e}_{\mathscr{C}} t : A \qquad \Gamma \vdash^{e}_{\mathscr{C}} A \equiv_{\beta} B : s}{\Gamma \vdash^{e}_{\mathscr{C}} t : B}\ \textit{Red}$$

Equivalence from implicit to explicit conversion

$$\Gamma \vdash_{\mathscr{C}} t : A \Leftrightarrow \Gamma \vdash^{e}_{\mathscr{C}} t : A$$

Strange Loop 3

Let's try to prove the equivalence (conversion case):

$$\dfrac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \equiv_{\beta} B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \textit{Conv}$$

Strange Loop 3

Let's try to prove the equivalence (conversion case):

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \equiv_\beta B}{\Gamma \vdash_{\mathscr{C}} t : B}\ \mathit{Conv}$$

$$\frac{\Gamma \vdash^{e}_{\mathscr{C}} t : A \qquad \Gamma \vdash^{e}_{\mathscr{C}} B : s}{\quad}$$

Strange Loop 3

Let's try to prove the equivalence (conversion case):

$$\Gamma \vdash^{e}_{\mathscr{C}} t : A$$

$$\Gamma \vdash^{e}_{\mathscr{C}} B : s$$

$$\Gamma \vdash_{\mathscr{C}} t : A$$

We cannot use subject reduction on $\Gamma \vdash_{\mathscr{C}} B : s$

Strange Loop 3

Let's try to prove the equivalence (conversion case):

$$\frac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \equiv_\beta B}{\Gamma \vdash_{\mathscr{C}} t : B}\ Conv \qquad\qquad \frac{\Gamma \vdash^{e}_{\mathscr{C}} t : A \qquad \Gamma \vdash^{e}_{\mathscr{C}} B : s \qquad \Gamma \vdash^{e}_{\mathscr{C}} A \equiv_\beta B : s}{\Gamma \vdash^{e}_{\mathscr{C}} t : B}\ Conv^{e}$$

Instead, it would be easy if we had already proved the equivalence for the types ($\Gamma \vdash^{e}_{\mathscr{C}} A : s$ and $\Gamma \vdash^{e}_{\mathscr{C}} B : s$) thanks to subject reduction.

Strange Loop 3

Let's try to prove the equivalence (conversion case):

$$\dfrac{\Gamma \vdash_{\mathscr{C}} t : A \qquad \Gamma \vdash_{\mathscr{C}} B : s \qquad A \equiv_\beta B}{\Gamma \vdash_{\mathscr{C}} t : B}\ Conv$$

$$\dfrac{\Gamma \vdash^{e}_{\mathscr{C}} t : A \qquad \Gamma \vdash^{e}_{\mathscr{C}} B : s \qquad \Gamma \vdash^{e}_{\mathscr{C}} A \equiv_\beta B : s}{\Gamma \vdash^{e}_{\mathscr{C}} t : B}\ Conv^{e}$$

Levels

We are looking for a measure which is:

1. strictly decreasing from a term t to its type A
2. stable by β
3. stable by subtree

Levels

Lets denote $>_{\mathcal{D}}: \mathcal{D} \to \mathcal{D} \to \mathbb{P}$, a relation on derivation trees such that

1. $\dfrac{\Pi}{\Gamma \vdash_{\mathscr{C}} t : A} >_{\mathcal{D}} \dfrac{\Pi'}{\Gamma \vdash_{\mathscr{C}} A : s}$ $(A \notin \mathcal{S})$
2. $\dfrac{\Pi}{\Gamma \vdash_{\mathscr{C}} t : A} \geq_{\mathcal{D}} \dfrac{\Pi'}{\Gamma \vdash_{\mathscr{C}} t' : A}$ (if $t \hookrightarrow_{\beta} t'$)
3. $\dfrac{\Pi}{\Gamma \vdash_{\mathscr{C}} t : A} \geq_{\mathcal{D}} \dfrac{\Pi'}{\Gamma' \vdash_{\mathscr{C}} u : B}$ (if Π' is a subtree of Π)

Theorem
The existence of $>_{\mathcal{D}}$ implies a measure function $\mathcal{L} : \mathcal{D} \to \mathcal{O}$ where $\mathcal{O}$ is a well-ordered set.

Levels are nice

Theorem
If $>_{\mathcal{D}}$ exists, then we have the correctness of the CTS encoding into Dedukti

Theorem
If $>_{\mathcal{D}}$ exists, then we have expansion postponement

Theorem
If $>_{\mathcal{D}}$ exists, then we have the equivalence between the implicit and the explicit conversion

Proof of expansion postponement with levels

Theorem
The existence of $>_{\mathcal{D}}$ implies expansion postponement:

$$\Gamma \vdash_{\mathscr{C}} t : A \Leftrightarrow \exists A', A \hookrightarrow_{\beta}^{*} A' \wedge \Gamma \vdash_{\mathscr{C}}^{r} t : A'$$

Proof.
By induction given by the measure function $\mathcal{L}$.

- Base case is trivial (though an induction on the derivation tree is needed).
- Inductive case is proved by induction on the derivation tree.

□

Proving the inductive case

Assuming expansion postponement at level o', we want to prove expansion postponement at level o (where $o >_{\mathcal{D}} o'$):

$$\frac{\Gamma, x : A \vdash_{\mathscr{C}} t : B \qquad \Gamma \vdash_{\mathscr{C}} (x : A) \to B : s}{\Gamma \vdash_{\mathscr{C}} \lambda x : A.\, t : (x : A) \to B}\ \lambda \qquad\qquad \frac{\Gamma, x : A \vdash^{r}_{\mathscr{C}} t : B'}{\Gamma \vdash^{r}_{\mathscr{C}} \lambda x : A.\, t : (x : A) \to B}\ \lambda^{r}$$

Proving the inductive case

Assuming expansion postponement at level o', we want to prove expansion postponement at level o (where $o >_{\mathcal{D}} o'$):

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- $\mathcal{L}(\Gamma \vdash_{\mathscr{C}} (x : A) \to B : s) = o_1$ with $o >_{\mathcal{D}} o_1$

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- $\mathcal{L}(\Gamma \vdash_{\mathscr{C}} (x : A) \to B : s) = o_1$ with $o >_{\mathcal{D}} o_1$
- $\mathcal{L}(\Gamma \vdash_{\mathscr{C}} (x : A) \to B' : s) \leq_{\mathcal{D}} o_1$ from second condition of $>_{\mathcal{D}}$

Proving the inductive case

Assuming expansion postponement at level o', we want to prove expansion postponement at level o (where $o >_{\mathcal{D}} o'$):

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- $\mathcal{L}(\Gamma \vdash_{\mathscr{C}} (x : A) \to B : s) = o_1$ with $o >_{\mathcal{D}} o_1$
- $\mathcal{L}(\Gamma \vdash_{\mathscr{C}} (x : A) \to B' : s) \leq_{\mathcal{D}} o_1$ from second condition of $>_{\mathcal{D}}$
- $\Gamma \vdash^{r}_{\mathscr{C}} (x : A) \to B' : s$ by EP

The big question

Is it possible to find an order $>_{\mathcal{D}}$?

Give it a try!

Instead of giving an order $>_{\mathcal{D}}$, we annotate a judgment with a level.

$$\frac{\Gamma, x : A \vdash_{\mathscr{C}}^{n+1} t : B \qquad \Gamma \vdash_{\mathscr{C}}^{n} (x : A) \to B : s}{\Gamma \vdash_{\mathscr{C}}^{n+1} \lambda x : A.\, t : (x : A) \to B} \; \lambda$$

$$\frac{\Gamma \vdash_{\mathscr{C}}^{n} f : (x : A) \to B \qquad \Gamma \vdash_{\mathscr{C}}^{n} a : A}{\Gamma \vdash_{\mathscr{C}}^{n} f\ a : B\{x \leftarrow a\}} \; app$$

A counterexample

In the context Γ:

- $Nat : \star$ (at level 1)
- $Vec : Nat \to \star$ (at level 2)
- $l : (x : Nat) \to Vec\ x$ (at level 3)

Assume we have derivation of (3 is the minimum level)

$$\Gamma \vdash^{3}_{\mathscr{C}} 10 : Nat$$

one can derive that

$$\Gamma \vdash^{3}_{\mathscr{C}} (\lambda x : Nat.\ l\ x)\ 10 : Vec\ 10$$

However, there is no derivation of

$$\Gamma \vdash^{2}_{\mathscr{C}} Vec\ 10 : \star$$

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In the context Γ:

- $Nat : \star$ (at level 1)
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- $l : (x : Nat) \to Vec\ x$ (at level 3)

As

Levels are not stable by substitution!

one can derive that

$$\Gamma \vdash^3_{\mathscr{C}} (\lambda x : Nat.\ l\ x)\ 10 : Vec\ 10$$

However, there is no derivation of

$$\Gamma \vdash^2_{\mathscr{C}} Vec\ 10 : \star$$

Other questions

- Is it possible to find an order $>_{\mathcal{D}}$? Hard!
- Is it possible to find an order $>_{\mathcal{D}}$ for some specification $\mathscr{C}$? easier! (ex: System Fω)
- Is it possible to find an order $>_{\mathcal{D}}$ for a concrete derivation tree in some specification $\mathscr{C}$? even easier!

Conclusion

- We have introduced levels
- It gives a natural solution to solve hard problems such as:
  - Expansion postponement
  - The equivalence between the explicit and implicit conversion
- The existence of levels is not guaranteed for all specifications $\mathscr{C}$

Future work

Conjecture 1
EP + termination implies the existence of $>_{\mathcal{D}}$.

Sufficient to prove the correctness of CTS encoding behind Coq,Agda,Lean in Dedukti

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Conjecture 1
EP + termination implies the existence of $>_{\mathcal{D}}$.

Sufficient to prove the correctness of CTS encoding behind Coq,Agda,Lean in Dedukti

Conjecture 2
$>_{\mathcal{D}}$ exists for every specification $\mathscr{C}$.

Idea

$$\frac{\Gamma \vdash^{n}_{\mathscr{C}} f : (x : A) \to B \qquad \Gamma \vdash^{n}_{\mathscr{C}} a : A}{\Gamma \vdash^{n}_{\mathscr{C}} f\ a : B\{x \leftarrow a\}}\ app$$

$$\Downarrow$$

$$\frac{\begin{array}{c}\Gamma \vdash^{n+1}_{\mathscr{C}} f : (x : A) \to B \\ \Gamma \vdash^{n+1}_{\mathscr{C}} a : A \qquad \Gamma \vdash^{n}_{\mathscr{C}} B\{x \leftarrow A\} : s\end{array}}{\Gamma \vdash^{n+1}_{\mathscr{C}} f\ a : B\{x \leftarrow a\}}\ app$$

- It is better (checked in practice)
- Not enough since cuts are not taken into account: the substitution is not applied on intermediate types